Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

14 May 2026

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Session number

2 hours

Instructions to students

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2226 – 5225

29 pages

Please **do not** write on this page.

Answers written on this page
will not be marked.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

In an experimental trial, 12 identical biased coins are made by coating the tails side with a heavy, metallic paint. When a biased coin is tossed, the probability that it lands heads side up is 0.73.

All 12 coins are tossed at the same time.

(a) Find the probability that exactly 8 coins will land heads side up. [2]

(b) Find the probability that more coins will land heads side up than tails side up. [2]

The experiment is repeated for a total of 49 trials.

(c) Find the expected number of trials in which more coins will land heads side up than tails side up. [2]

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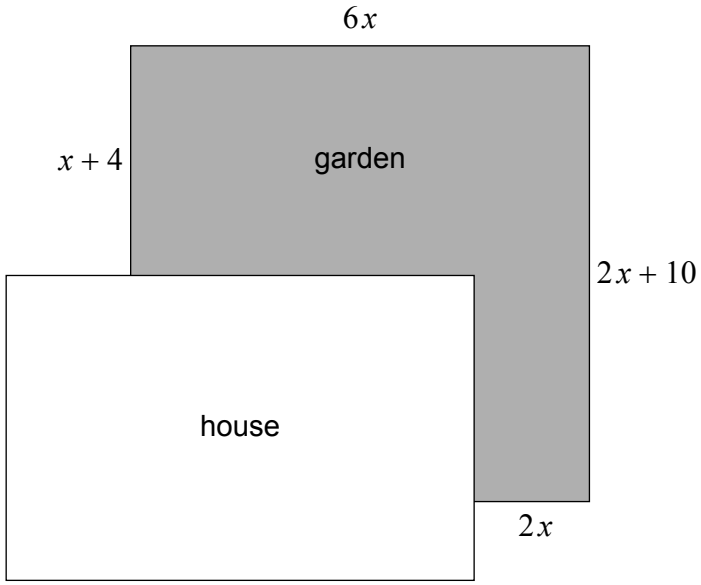
2. [Maximum mark: 7]

Jeff is designing a garden around a corner of his house. The lengths, in metres, of each side of the garden are given in terms of x, as shown in the following diagram.

diagram not to scale



(a) Show that the garden's area, $A\,\text{m}^2$, is given by $A = 8x^2 + 36x$. [3]

The garden's area is $72\,\text{m}^2$.

(b) Find the value of x. [2]

Jeff decides to construct a fence around the garden. The fence is not required around the parts of the garden that touch the side of the house.

(c) Find the length of fence that Jeff needs. [2]

(This question continues on the following page)

(Question 2 continued)

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3. [Maximum mark: 8]

The grasshopper population in a cornfield is modelled by the function P.

Scientists are developing a new pesticide to reduce the population of grasshoppers. They have found that the rate of change of the grasshopper population can be modelled by

$$P'(t) = 70 + t - 3t^2,$$

where t is the time, in weeks, after the pesticide was sprayed.

(a) (i) Find the value of $P'(5)$.

(ii) Interpret, in context, the value of $P'(5)$. [2]

Five weeks after the pesticide was sprayed, the grasshopper population in the cornfield was 254.

(b) Find an expression for $P(t)$. [4]

(c) Find the amount of time required to eliminate the grasshopper population in the cornfield. [2]

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4. [Maximum mark: 7]

An airline conducted a survey to see how many items of luggage are lost during transit. A sample of 80 items of luggage were selected at random and tracked, and 5 of those were lost.

Items of luggage that weigh over $18\,\text{kg}$ are considered "heavy". In the sample, 24 items of luggage were heavy, and 2 of those were lost.

(a) Using this information, complete the following table. [2]

	Lost	**Not lost**	**Totals**
Heavy			
Not heavy			
Totals	5		80

(b) An item of luggage is selected at random from the sample. Find the probability that this item is

(i) not heavy.

(ii) lost, given that it is heavy. [2]

It can be assumed that the proportions of lost luggage in the survey are representative of the proportions on any given flight with this airline.

The airline's research also shows that the weight of an item of luggage can be modelled by a normal distribution with a mean of $16\,\text{kg}$ and a standard deviation of $3\,\text{kg}$.

An item of luggage is selected at random from all those in transit with this airline.

(c) Find the probability that it will be heavy and lost. [3]

(This question continues on the following page)

(Question 4 continued)

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5. [Maximum mark: 5]

A directed graph, G, has an adjacency matrix, $\boldsymbol{M}$, where

$$\boldsymbol{M} = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

(a) Write down one directed edge in G. [1]

It is given that

$$\boldsymbol{M}^2 = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\ \begin{pmatrix} 2 & 0 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 & 1 \\ 0 & 2 & 2 & 1 & 0 \\ 2 & 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array} \text{ and } \boldsymbol{M}^3 = \begin{array}{c} \\ \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\ \begin{pmatrix} 1 & 4 & 4 & 2 & 1 \\ 4 & 2 & 4 & 5 & 2 \\ 4 & 1 & 3 & 4 & 2 \\ 1 & 4 & 4 & 2 & 1 \\ 2 & 0 & 1 & 2 & 1 \end{pmatrix} \end{array}.$$

(b) State, with justification, whether it is possible to move from any vertex to any other vertex in

(i) exactly three transitions.

(ii) at most three transitions. [4]

(This question continues on the following page)

(Question 5 continued)

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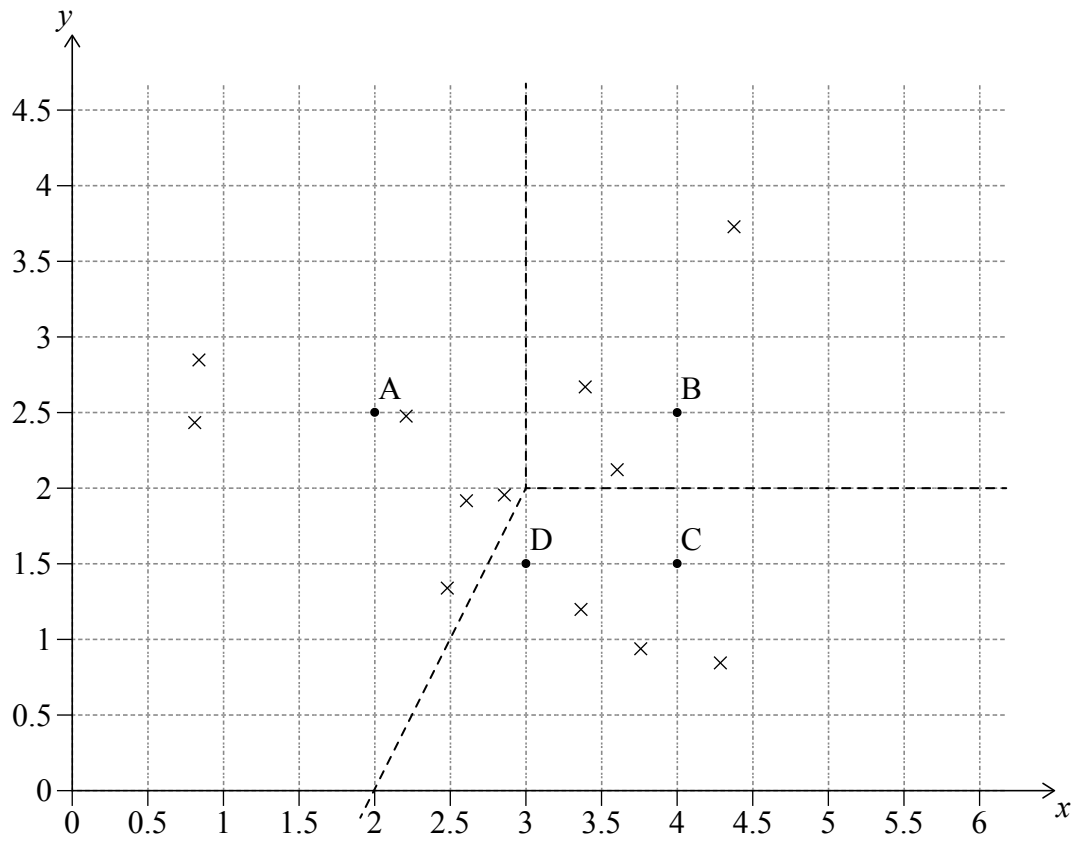
6. [Maximum mark: 6]

George is planning to visit Rome. He finds a list of the top 12 tourist attractions and their positions on a map of the city.

George also finds four possible hotels to stay at. He decides he will book the hotel that is closest to most attractions.

Each tourist attraction is marked with an × on the following coordinate axes. The hotels are at the following points: $\mathrm{A}(2, 2.5)$, $\mathrm{B}(4, 2.5)$, $\mathrm{C}(4, 1.5)$ and $\mathrm{D}(3, 1.5)$. George draws a Voronoi diagram with A, B and C as the sites. The edges are shown as dashed lines on the diagram.



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(Question 6 continued)

(a) Draw the perpendicular bisectors of $[AD]$, $[BD]$ and $[CD]$ on the diagram using three new dashed lines. [3]

(b) Hence complete the Voronoi diagram with the four hotels as the sites. Represent the edges of the Voronoi diagram as solid lines. [2]

(c) State which of the hotels is closest to most attractions. [1]

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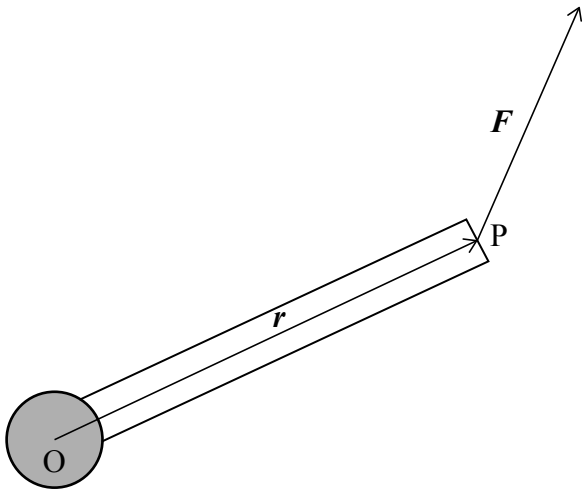
7. [Maximum mark: 7]

Consider the vectors $\boldsymbol{r} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$ and $\boldsymbol{F} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$.

(a) Find the angle between $\boldsymbol{r}$ and $\boldsymbol{F}$. [4]

A lever is attached to a fixed pivot at point O at the centre of a coordinate system.

Force $\boldsymbol{F} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ acts on the lever at point P with position vector $\boldsymbol{r} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$.



The turning effect of the force produced at this instant, $\boldsymbol{T}$, is defined as $\boldsymbol{T} = \boldsymbol{r} \times \boldsymbol{F}$.

(b) Find $|\boldsymbol{T}|$. [3]

(This question continues on the following page)

(Question 7 continued)

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8. [Maximum mark: 8]

Material is produced in lengths of 10 metres and formed into rolls. The number of flaws in the material can be modelled by a Poisson distribution with a mean of 0.21 flaws per metre.

(a) Find the probability that there are

(i) no flaws in a 1-metre length of the material.

(ii) at most 2 flaws in a 10-metre roll. [3]

The total cost of checking and correcting the flaws is 5 dollars per roll and an additional 12 dollars per flaw. Let the total cost of checking and correcting the flaws in a single roll be C.

(b) Find

(i) $\mathrm{E}(C)$.

(ii) $\mathrm{Var}(C)$. [4]

(c) Give one reason why C cannot be modelled by a Poisson distribution. [1]

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9. [Maximum mark: 8]

A coupled system of differential equations is given below.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x + 2y$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = 5x + 4y$$

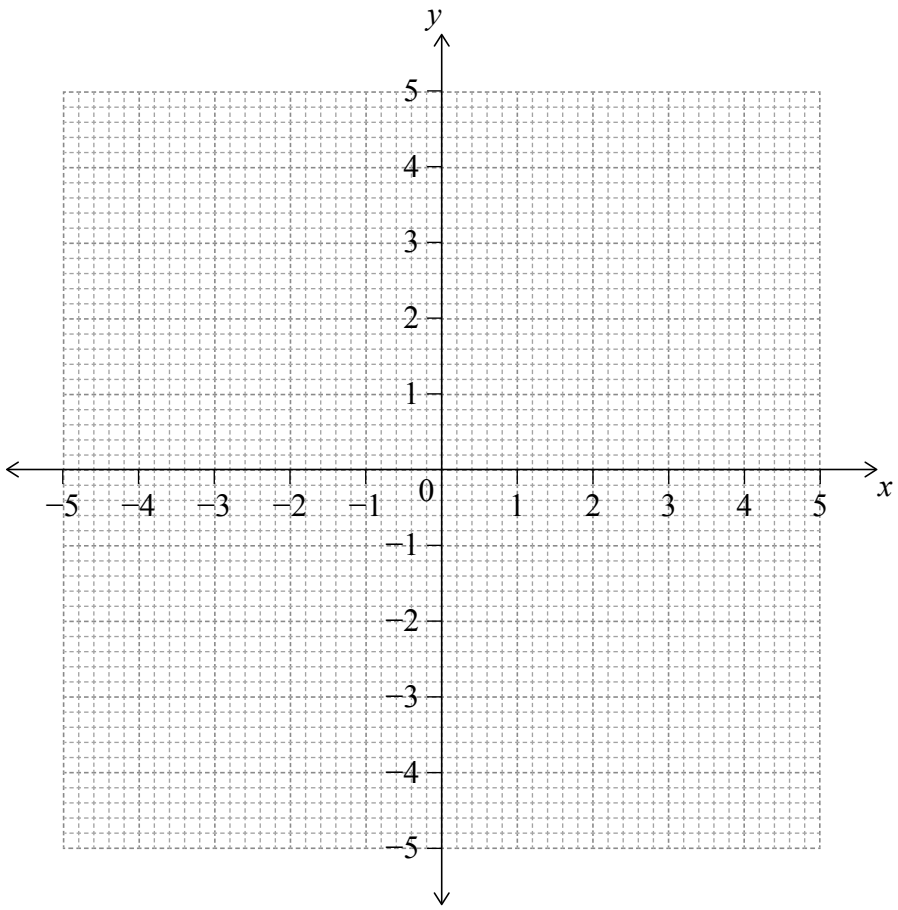
The system has an eigenvalue of -1 with a corresponding eigenvector of $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

(a) Find the second

(i) eigenvalue.

(ii) eigenvector. [5]

(b) Sketch the phase portrait for the system on the following set of axes. Your sketch should include the lines defined by the eigenvectors, the direction of motion along these lines, and one trajectory in each of the four sections of the phase portrait. [3]



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(Question 9 continued)

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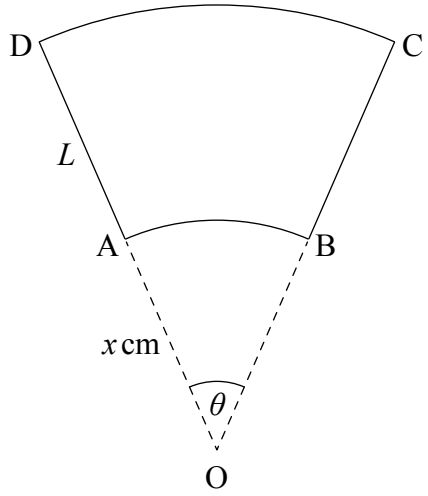
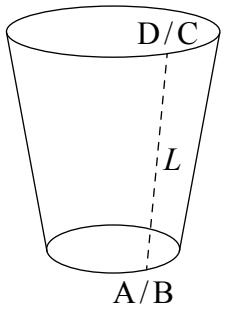
10. [Maximum mark: 7]

The curved surface of a coffee cup is made from a flat piece of card, ABCD, by joining points A to B and D to C. L is the slant height of the cup. AB and DC are both arcs of a circle centred at O.

Let the distance $\mathrm{OA} = x$ and $\mathrm{A\hat{O}B} = \theta$. This is shown in the following diagrams.

diagram not to scale





The circumference of the top rim of the cup is $24\,\mathrm{cm}$, the circumference of the base is $18\,\mathrm{cm}$ and $L = 10\,\mathrm{cm}$.

(a) Find the value of x and the value of θ, giving your answer for θ in radians. [5]

(b) Find the curved surface area of the outside of the coffee cup. [2]

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11. [Maximum mark: 7]

The function f is defined as $f(x) = 4 - x^2$, $0 \leq x \leq 2$.

(a) Find the inverse function $f^{-1}(x)$, clearly stating its domain. [4]

The curve $y = f(x)$ is rotated through $360°$ about the y-axis.

(b) Find the volume of the solid formed. [3]

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12. [Maximum mark: 5]

Hailey is playing a game in which a die is rolled. Each time Hailey rolls the die she receives the number she rolls as points, but if she rolls a 2 she instead loses all her points.

The following table shows the distribution of Y, where Y is the amount of points Hailey could receive when she already has x points.

Number on the die	1	2	3	4	5	6
Points received (y)	a	b	3	4	5	6
$\mathbf{P}(Y=y)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

(a) Write down

(i) the value of a.

(ii) an expression for b in terms of x. [2]

(b) Find an expression for $\mathrm{E}(Y)$. [2]

Before each roll of the die, Hailey has the choice to stop playing the game and keep all her points, or to continue playing and risk losing all her points.

Hailey decides to stop when $\mathrm{E}(Y) < 0$.

(c) Find the least number of points Hailey could have when she decides to stop. [1]

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13. [Maximum mark: 5]

A particle has velocity $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 2\sin(3t) - 1 \\ 2t \end{pmatrix}$, for $t \geq 0$.

(a) Find the value of t at which the particle is first travelling parallel to the y-axis. [2]

At $t = 0$ the particle is at $(0, 0)$.

(b) Find an expression for the particle's position, $\begin{pmatrix} x \\ y \end{pmatrix}$, at time $t (t \geq 0)$. [3]

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14. [Maximum mark: 8]

Consider the differential equation $\frac{dy}{dx} = 3x^2 + 2y - 6.75$.

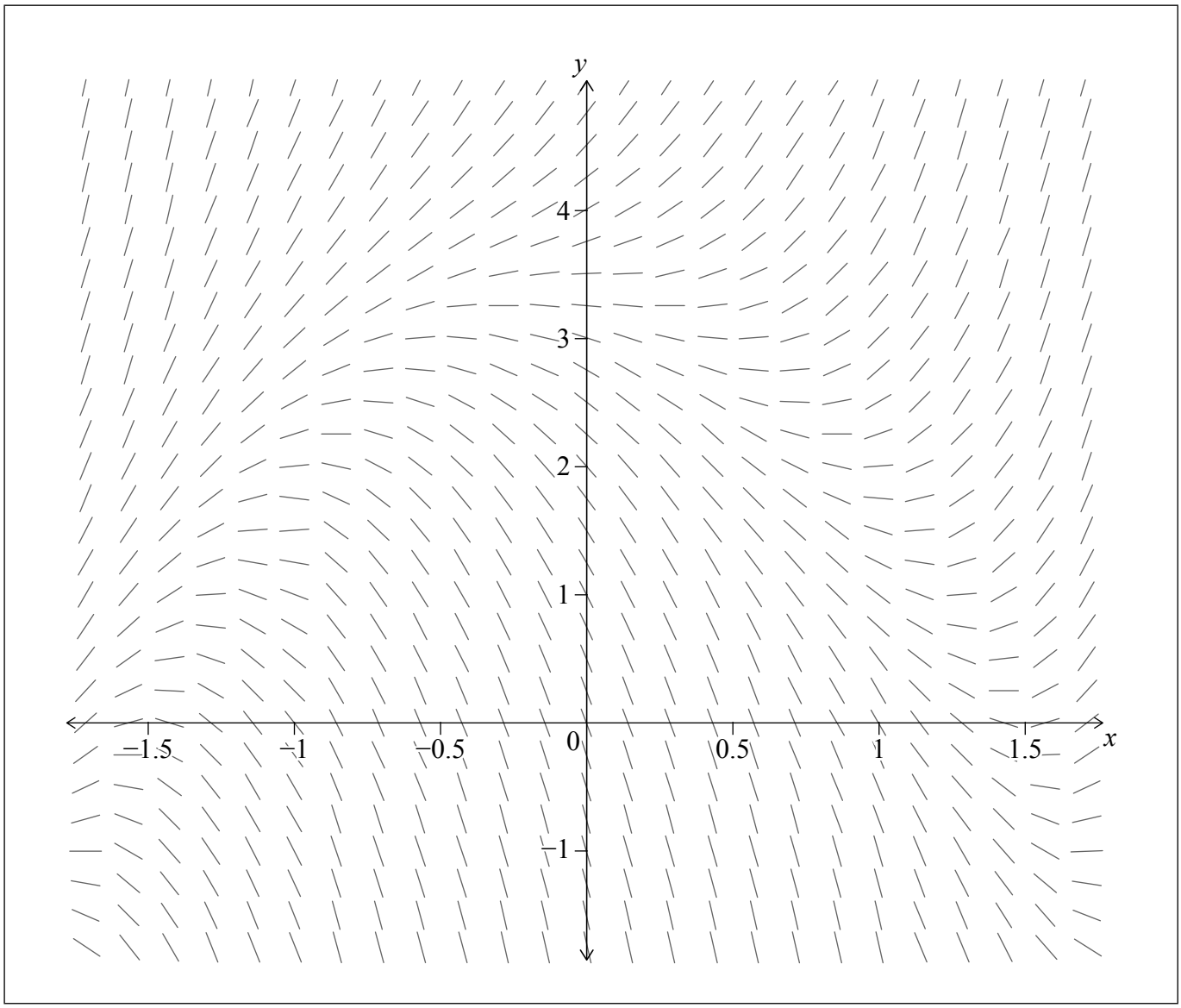
A particular solution, $y = f(x)$, has a local maximum at $x = -0.5$.

(a) Show that the graph of $y = f(x)$ passes through the point $(-0.5, 3)$. [2]

The graph of $y = f(x)$ has a minimum point at approximately $x = 1.6$.

(b) Use Euler's method with a step length of 0.1 to find an estimate for the value of y when $x = 1.6$. [3]

(c) Sketch the graph of $y = f(x)$ on the following slope field diagram. [1]



(d) On the same axes, sketch the graph of the solution to the differential equation that passes through the point $(0, 4)$. [2]

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(Question 14 continued)

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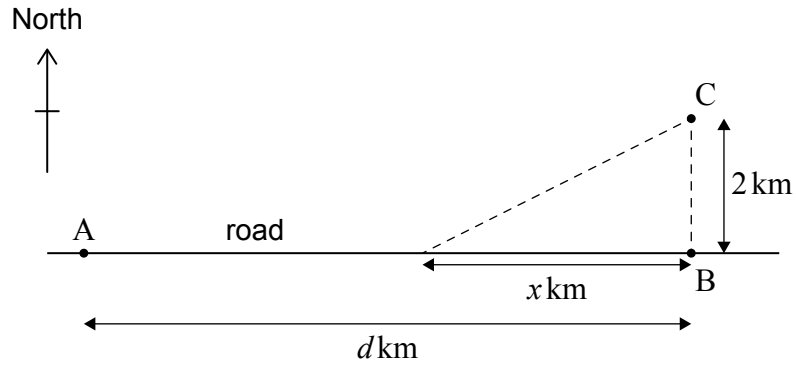
15. [Maximum mark: 9]

A road runs from east to west. Some houses are being built at point C, $2\,\mathrm{km}$ due north of point B on the road. These houses need to be connected to the main electricity grid with a length of cable. An engineer is trying to calculate the cost of connecting the cable to the houses from point A on the road, which is a distance $d\,\mathrm{km}$ due west of point B.

The engineer considers installing the cable along the road until a point $x\,\mathrm{km}$ from B and from there installing the cable directly to C.

This information is shown in the following diagram.



The cost to install the cable along the road is $\$2000$ per kilometre and to install the cable across the land north of the road is $\$10\,000$ per kilometre.

The total cost of installing the cable, in thousands of dollars, T, is therefore

$$T = 2(d - x) + 10\sqrt{4 + x^2}\,.$$

(a) (i) Find $\dfrac{\mathrm{d}T}{\mathrm{d}x}$.

(ii) Solve $\dfrac{\mathrm{d}T}{\mathrm{d}x} = 0$. [5]

It is given that $\dfrac{\mathrm{d}^2T}{\mathrm{d}x^2} = \dfrac{40}{\left(4 + x^2\right)^{\frac{3}{2}}}$.

(b) Hence or otherwise, verify that the value found in part (a)(ii) corresponds to a minimum value for T. [2]

Let L be the minimum cost, in thousands of dollars, for installing the cable.

(c) Find an expression for L in the form $L = pd + q$, where $p, q \in \mathbb{R}$. [2]

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(Question 15 continued)

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16. [Maximum mark: 7]

Let the complex number $z = 3e^{2i}$ and the complex number $w = z + (6 + 2.5i)$.

(a) Find $|w - z|$. [2]

(b) Find w in the form $re^{\theta i}$, where $r, \theta > 0$. [2]

The numbers z, w and 0 are represented in an Argand diagram by the points A, B and C respectively.

(c) Find $A\hat{B}C$. [3]

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